INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust Nonlinear Control 2017; **27**:1690–1705
Published online 2 August 2016 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/rnc.3606

Distributed model predictive control for multi-agent flocking via neighbor screening optimization

Lifeng Zhou and Shaoyuan Li*,†

Department of Automation, Shanghai Jiao Tong University, Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai 200240, China

SUMMARY

This study presents a distributed model predictive control (MPC) strategy to achieve flocking of multi-agent systems. Based on the relative motion between each pair of neighboring agents, we introduce a neighbor screening protocol, by which each agent only focuses on its neighbors, which have the relative motion that violates the formation of flocks. Then, a truly distributed MPC flocking algorithm (Algorithm 1) is designed with consideration of neighbor screening mechanism. Specifically, at each sampling instant, each agent monitors the information in the networked system, finds its neighbors to form its subsystem, determines the screened neighbor set, and optimizes its plan by collecting the position states within the screened subsystem. And geometric properties of the optimal path are used to guarantee the formation of the flock without inter-agent collision. Finally, the performance and advantage of the proposed distributed MPC flocking strategy are vividly verified by the simulation results. Copyright © 2016 John Wiley & Sons, Ltd.

Received 28 February 2016; Revised 6 June 2016; Accepted 21 June 2016

KEY WORDS: distributed model predictive control (MPC); multi-agent systems (MASs); flocking; neighbor screening

1. INTRODUCTION

In recent years, more and more efforts from diverse fields including biology, physics, social sciences, and computer science have been devoted to the collective behaviors of multi-agent systems (MASs), which have the remarkably characteristic patterns such as flocking, swarming, and schooling from individual interactions [1–10]. Exploring the mechanisms of these fascinating behaviors by using distributed or decentralized decision-making approaches has significant implications on mobile sensor network, collaborative robots, ground/underwater vehicles, unmanned aerial vehicles (UAVs), satellite cluster alignment, and congestion alleviation of communication networks [11–16].

In 1986, Reynolds [1] has pioneered three fundamental rules for animation of natural flocks/swarms: separation, alignment, and cohesion. Based on these three basic rules, a surge of brilliant achievements have been obtained for the flocking of MASs. Vicsek *et al.* [2] have introduced the famous Vicsek flocking model based on the inter-agent velocity regulation. Later, Jadbabaie *et al.* [3] have extended the flocking to several general models. As a representative work, Olfati-Saber [8] has presented three flocking algorithms for design and analysis of flocking problem, which consider flocking behaviors in free-space and presence of multi-obstacle environment. Generally, a flocking control strategy is constructed by combining the gradient of a collective potential or cost function which penalizes the deviation from the flocking structure with a velocity synchronization approach which achieves alignment by coordinating inter-agent velocity [3–8].

*Correspondence to: Shaoyuan Li, Department of Automation, Shanghai Jiao Tong University, Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai 200240, China.
†E-mail: syli@sjtu.edu.cn

To date, in most of previous flocking protocols, each agent is only available to collect the information of its neighbors at the given time, and then takes a timely motion decision without taking advantage of some predictive intelligence. However, biology literature has witnessed that almost all living creatures have predictive intelligence allowing them to predict the future behaviors of their neighbors based on the current and past observations. Examples about creatures' predictive intelligence have been vividly exhibited in the swarming of bees [17, 18], biological system [19–21], and so on. Such predictive mechanisms [22–24] with capacities of optimizing relevant performance and taking account of the constraints have attracted researchers to apply these appealing features in the study of MASs.

To make use of individual predictive intelligence among collective behaviors, Ferrai-Trecate *et al.* [25] have proposed a decentralized MPC strategy to achieve the consensus of the MASs with control input constraints. Following this line, Zhang *et al.* [26–30] have designed a series of MPC methods for the collective behaviors of MASs, which improve the efficacy of the consensus by predictive mechanism and pinning control [31], deal with the control input constraints, and achieve the flocks of second-order MASs. A general framework for distributed MPC of discrete-time nonlinear systems has been presented in [32], in which cooperative tasks are vividly illustrated by synchronization of four identical Van der Pol oscillators. For sampled-data MASs, Zhan and Li [33] have presented a distributed MPC consensus algorithm, achieved the fast weighted-average consensus of MASs and enlarged the feasible range of sampling interval. And the flocking control of MASs via MPC has been researched in their work [34] where both centralized and distributed impulsive MPC flocking algorithms are proved to realize the flocks of MASs based on position-only measurements. Recently, we have proposed several distributed MPC strategies [35, 36] to achieve the consensus or synchronization of MASs, which are displayed by five unmanned aerial vehicle model and five linear oscillators, respectively.

In short, the flocking problem focuses on achieving the flocks or a rigid conformation where each pair of neighboring agents has the same distance. Obviously, only when the velocities of all agents reach the consensus or synchronization, can the rigid flocks be guaranteed. Therefore, as two representative themes of MASs, flocking, and consensus have a close connection with each other. However, designing a distributed MPC scheme with optimal energy and less local information exchange to form the rigid flocks for MASs is still a challengeable work. In this paper, our main contributions include (i) A neighbor screening protocol, based on the relative motion between each pair of neighboring agents, is proposed to screen agent's neighbor(s) to determine screened neighbor set for receding horizon optimization in MPC. The proposed neighbor screening mechanism is inspired by the work [37] where each agent only considers the subset of its neighbor agents with whom it is susceptible to collide. (ii) A truly distributed MPC flocking algorithm, that is, at each sampling instant, each agent monitors the information in the networked system, finds its neighbors, determines its screened neighbor set, and collects the position states in the screened subsystem for optimization, is presented. (iii) The advantages including less local information exchange and fast convergence of proposed distributed MPC flocking strategy are illustratively displayed.

The remainder of this paper is organized as follows. The necessary notations and definitions, basic knowledge about graph theory and MPC flocking control, and our proposed neighbor screening mechanism are presented in Section 2. In Section 3, we propose a distributed MPC flocking strategy via neighbor screening to achieve the lattice conformation. Simulation results showing the performance of our proposed distributed MPC flocking strategy are displayed in Section 4. Finally, Section 5 summarizes the paper.

2. PRELIMINARIES

We first introduce some notations and definitions to be used throughout this paper. $\boldsymbol{1} = [1, 1, ..., 1]^T$, $\boldsymbol{e}_2 = [1\ 0]_{1\times 2}$. Matrix $\boldsymbol{I}_m$ indicates the identify matrix with dimension m. The definition $\mathbb{R}^m$ denotes the set of m dimensional real column vectors. $\|\cdot\|$ and $\otimes$ indicate the Euclidean norm and the Kronecker product, respectively. The notation $*(k+t|k)$ denotes the predicted value $*$ at instant $k+t$ based on the currently available information at instant k.

DOI: 10.1002/rnc

The fundamental concepts of flocking for MASs in [8], our proposed neighbor screening mechanism and basic knowledge of MPC flocking method are described in the following.

2.1. Flocks: proximity nets and α-lattices

Consider a group of N agents. It can be abstractly described as a graph indicated by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, consisting of a set of vertices $\mathcal{V} = \{1, 2, ..., N\}$ and edges $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, j \neq i\}$. $|\mathcal{V}|$ and $|\mathcal{E}|$ denote the order and the size of the graph, respectively. If $(i, j) \in \mathcal{E} \iff (j, i) \in \mathcal{E}$, we call the graph $\mathcal{G}$ is undirected, and this paper concerns undirected graph only. If there exists a path, that is, a sequence of edges $(i, k_1), (k_1, k_2), ..., (k_{s-1}, k_s), (k_s, j), k_t \in \mathcal{V}, t = 1, ..., s$ between any two vertices $i, j \in \mathcal{V}$, the undirected graph $\mathcal{G}$ is connected. The neighbor set of agent i is denoted by $N_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$, and $|N_i|$ denotes the number of i' neighbors. The motion of each agent $i, i \in \{1, ...N\}$ is governed by the kinematics

$$\begin{aligned} \dot{\boldsymbol{q}}_i &= \boldsymbol{p}_i \\ \dot{\boldsymbol{p}}_i &= \boldsymbol{u}_i, \end{aligned} \tag{1}$$

where $\boldsymbol{q}_i, \boldsymbol{p}_i, \boldsymbol{u}_i \in \mathbb{R}^m$ (e.g., $m = 2, 3$) indicate the position, velocity, and acceleration of agent $i, i \in \mathcal{V}$, respectively. For notational convenience, $\forall i, j \in \mathcal{V}$, define

$$\begin{aligned} \boldsymbol{q}_{ji} &= \boldsymbol{q}_i - \boldsymbol{q}_j, \\ \boldsymbol{p}_{ji} &= \boldsymbol{p}_i - \boldsymbol{p}_j. \end{aligned} \tag{2}$$

Next, define $r_c > 0$ as the communication range between two agents. Then the neighbor set of agent i is

$$N_i = j \in \mathcal{V} : d_{ij} < r_c, \tag{3}$$

where $d_{ij} := \|\boldsymbol{q}_j - \boldsymbol{q}_i\|$ denotes the Euclidean distance between agent i and agent j. And then the set of edges between neighbors is defined as

$$\mathcal{E}(\boldsymbol{q}) = (i, j) \in \mathcal{V} \times \mathcal{V} : d_{ij} < r_c, i \neq j \tag{4}$$

that depends on the organization of all agents $\boldsymbol{q} = col(\boldsymbol{q}_1, \boldsymbol{q}_2, ..., \boldsymbol{q}_N) \in \mathbb{R}^{Nm}$. The topology $G(\boldsymbol{q}) = (\mathcal{V}, \mathcal{E}(\boldsymbol{q}))$ is called a *proximity net*, and the configuration $(G(\boldsymbol{q}), \boldsymbol{q})$ is called a *proximity structure*.

In [8], Olfati-Saber proposed an α-lattice conformation to model a desirable geometry of flocks, which can be described as the solutions of the following algebraic constraints:

$$d_{ij} = d, \ \forall j \in N_i(\boldsymbol{q}), \tag{5}$$

where d indicates the desirable distance between neighboring agents, and $d < r_c$. In order to discribe the structure, which is close to the α-lattice, Olfati-Saber also shown the following set of inequalities:

$$-\delta \leqslant d_{ij} - d \leqslant \delta, \ \forall j \in N_i(\boldsymbol{q}), \tag{6}$$

and defined its solution as a quasi α-lattice. Here, δ denotes the allowable error.

Besides, with the formation of α-lattice structure, the velocity consensus of all the agents should be reached

$$\boldsymbol{p}_i = \bar{\boldsymbol{p}}, \ \forall i \in \mathcal{V}, \tag{7}$$

with the average velocity of all agents $\bar{\boldsymbol{p}} = 1/N \sum_{i=1}^{N} \boldsymbol{p}_i$.

DOI: 10.1002/rnc

2.2. Neighbor screening via relative motion

Our main objective is to realize the α-lattice conformation where each pair of neighboring agents has a desirable distance d. If the relative motion between a pair of neighboring agents facilitates the arrival of the desirable conformation, it is not necessary to take action to regulate their behaviors. On the contrary, if the relative motion contradicts the formation of the desirable lattice, some strategies should be conducted to constrain this trend. Thus, as for each agent i, we screen its neighbor(s) $j \in N_i$ and only focus on the neighbors with the relative motion, which contradicts the desirable conformation, and collect these neighbors in a set N_i^-. We first display the relative motion of a pair of neighbors, i and j, in Figure 1.

And define the relative motion index between agent i and its neighbor j as

$$M_{ij} = M_i - M_j, \tag{8}$$

where $M_i := \boldsymbol{q}_{ji}^T \boldsymbol{p}_i$ and $M_j := \boldsymbol{q}_{ji}^T \boldsymbol{p}_j$ indicate the motion indices for agent i and j, respectively. Then the following cases are provided to obtain the screened neighbor set N_i^-:

Case 1, $d_{ij} < d < r_c$:

This case shows that the distance between agent i and its neighbor(s) j is shorter than the desirable distance d, so its neighbor(s) with the relative motion, which contributes towards reducing the distance, should be chosen. Then consider the following subcases:

(1) $M_{ij} > 0$. It implies that the relative motion between agent i and agent j contributes towards increasing the distance, and thus $j \notin N_i^-$.
(2) $M_{ij} < 0$. It implies that the relative motion between agent i and agent j contributes towards reducing the distance, and thus $j \in N_i^-$.
(3) $M_{ij} = 0$. It implies no relative motion between agent i and agent j. But the distance $d_{ij} \neq d$, and thus $j \in N_i^-$.

Case 2, $d < d_{ij} < r_c$:

This case shows that the distance between agent i and its neighbor(s) j is longer than the desirable distance d, so its neighbor(s) with the relative motion, which contributes towards increasing the distance, should be chosen. Then consider the following subcases:

(1) $M_{ij} > 0$. It implies that the relative motion between agent i and agent j contributes towards increasing the distance, and thus $j \in N_i^-$.
(2) $M_{ij} < 0$. It implies that the relative motion between agent i and agent j contributes towards reducing the distance, and thus $j \notin N_i^-$.
(3) $M_{ij} = 0$. It implies no relative motion between agent i and agent j. But the distance $d_{ij} \neq d$, and thus $j \in N_i^-$.

Case 3, $d_{ij} = d < r_c$:

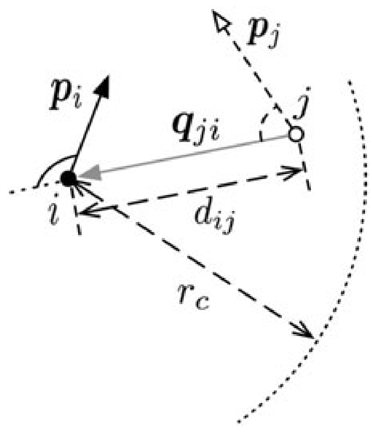

Figure 1. Relative motion of a pair of neighboring agents.

DOI: 10.1002/rnc

This case shows that the distance between agent i and its neighbor(s) j is equal to the desirable distance d, so its neighbor(s) with the relative motion should be chosen. Then consider the following subcases:

(1) $M_{ij} \neq 0$. It implies that there exists the relative motion between agent i and agent j, and thus $j \in N_i^-$.
(2) $M_{ij} = 0$. It implies no relative motion between agent i and agent j, and thus $j \notin N_i^-$.

2.3. *Flocking via model predictive control*

Consider that MPC involves the state prediction of discrete future time instants, we discretise (1) as

$$\begin{aligned} \boldsymbol{q}_i(k+1) &= \boldsymbol{q}_i(k) + T\boldsymbol{p}_i(k) \\ \boldsymbol{p}_i(k+1) &= \boldsymbol{p}_i(k) + T\boldsymbol{u}_i(k), \end{aligned} \tag{9}$$

where T indicates sampling interval. For notational convenience, define $\boldsymbol{x}_i(k) := \left[\boldsymbol{q}_i^T(k), \boldsymbol{p}_i^T(k)\right]^T \in \mathbb{R}^{2m}$.

Through predicting future states in H_p steps based on the dynamic (9) and optimizing the cost function (10), the control inputs can be computed in future H_u steps.

The objective of flocking control is to achieve an α-lattice conformation, namely, satisfying (5). Thus, define the cost function as

$$\min_{\{\boldsymbol{u}(k+t|k)\}_{t=0}^{H_u-1}} J(k) = \sum_{t=1}^{H_p}\left(\sum_{i,j\in\mathcal{V}} \|d_{ij} - d\|^2\right) + \lambda \sum_{t=0}^{H_u-1} \|\boldsymbol{u}(k+t|k)\|^2, \tag{10}$$

where $*(k+t|k)$ indicates the prediction of sampling instant $k+j$ at current sampling instant k, and $\boldsymbol{u} = col(\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_N)$. H_p and H_u denote prediction horizon and control horizon, respectively. At current sampling instant k, agents collect the global state information, collaboratively optimize the cost function (10) to obtain $\{\boldsymbol{u}^*(k+t|k)\}_{t=0}^{H_u-1}$, and apply $\boldsymbol{u}^*(k|k)$ only.

3. FLOCKING VIA DISTRIBUTED MODEL PREDICTIVE CONTROL ALGORITHM

The MPC flocking strategy presented Section 2.3 is a centralized MPC method, where the states of global system are known to each agent. However, due to agents' limited communication capabilities in actual situation, they usually have the access to some information provided by their neighbors instead of the global knowledge. Thus, we further design a distributed MPC flocking strategy, in which each agent optimizes its control input by receiving the state information of its neighbors and itself only.

3.1. *Prediction model and subsystem design*

First, rewrite the discrete-time dynamics (9) of agent i in a compact way

$$\boldsymbol{x}_i(k+1) = \boldsymbol{A}\boldsymbol{x}_i(k) + \boldsymbol{B}\boldsymbol{u}_i(k), \tag{11}$$

with

$$\boldsymbol{A} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \boldsymbol{I}_m, \ \boldsymbol{B} = \begin{bmatrix} 0 \\ T \end{bmatrix} \otimes \boldsymbol{I}_m.$$

Through iterative computations of (11), we obtained the prediction model of agent i

$$\boldsymbol{X}_i(k+1) = \boldsymbol{P}_x\boldsymbol{x}_i(k) + \boldsymbol{P}_u\boldsymbol{U}_i(k), \tag{12}$$

DOI: 10.1002/rnc

where $\boldsymbol{X}_i(k+1) := \left[\boldsymbol{x}_i^T(k+1|k), \boldsymbol{x}_i^T(k+2|k), \cdots, \boldsymbol{x}_i^T(k+H_p|k)\right]^T$, $\boldsymbol{U}_i(k) := \left[\boldsymbol{u}_i^T(k|k), \boldsymbol{u}_i^T(k+1|k), \cdots, \boldsymbol{u}_i^T(k+H_p-1|k)\right]$, $\boldsymbol{P}_x := col\left[\boldsymbol{A}^T, (\boldsymbol{A}^2)^T, \cdots, (\boldsymbol{A}^{H_p})^T\right]_{2mH_p\times 2m}$ and $\boldsymbol{P}_u =$

$$\begin{bmatrix} \boldsymbol{B} & & \\ \boldsymbol{AB} & \ddots & \\ \vdots & \vdots & \boldsymbol{B} \\ \boldsymbol{A}^{H_p-1}\boldsymbol{B} & \cdots & \boldsymbol{AB} \;\; \boldsymbol{B} \end{bmatrix}_{2mH_p\times mH_p},$$

where the control horizon H_u equals to the prediction horizon H_p.

Then, decompose the global networked system consisting of N agents into N subsystems accordingly. For agent i, its subsystem consists of its neighbors and itself, defined by $S_i := \left\{i, n_1^i, n_2^i, ..., n_{|N_i|}^i\right\}$. Thus, the state of S_i is indicated by $\boldsymbol{x}^i := \left[\boldsymbol{x}_i^T, \boldsymbol{x}_{i1}^T, \boldsymbol{x}_{i2}^T, \cdots, \boldsymbol{x}_{i_{|N_i|}}^T\right]^T$.

3.2. Optimization via neighbor screening

In the subsystem S_i, agent i only cares about its neighbor(s) in the screened neighbor set N_i^-, because these neighbors have the relative motion that violates desirable lattice conformation. Thus, define the screened subsystem for agent i as S_i^-, which consists of agent i and i' neighbor(s) in the screened neighbor set. The state and the position state of screened subsystem S_i^- are indicated by $\boldsymbol{x}^{i-} := \left[\boldsymbol{x}_i^T, \boldsymbol{x}_{i1}^T, \boldsymbol{x}_{i2}^T, \cdots, \boldsymbol{x}_{i_{|N_i^-|}}^T\right]^T$ and $\boldsymbol{q}^{i-} := \left[\boldsymbol{q}_i^T, \boldsymbol{q}_{i1}^T, \boldsymbol{q}_{i2}^T, \cdots, \boldsymbol{q}_{i_{|N_i^-|}}^T\right]^T$, respectively. Based on the state prediction rule in MPC, define $\boldsymbol{X}^{i-} := \left[\boldsymbol{X}_i^T, \boldsymbol{X}_{i1}^T, \cdots, \boldsymbol{X}_{i_{|N_i^-|}}^T\right]^T$ with $\boldsymbol{X}_j(k+1) := \left[\boldsymbol{x}_j^T(k+1|k), \boldsymbol{x}_j^T(k+2|k), \cdots, \boldsymbol{x}_j^T(k+H_p|k)\right]^T$, $j \in N_i^-$. Then, the future state of the subsystem S_i^- is

$$\boldsymbol{X}^{i-}(k+1|k) = \boldsymbol{P}_x^{i-}\boldsymbol{x}^{i-}(k) + \boldsymbol{P}_u^{i-}\boldsymbol{U}^{i-}(k), \tag{13}$$

with

$$\begin{aligned} \boldsymbol{P}_x^{i-} &= I_{|N_i^-|+1} \otimes \boldsymbol{P}_x \in \mathbb{R}^{2mH_p(|N_i^-|+1)\times 2m(|N_i^-|+1)}, \\ \boldsymbol{P}_u^{i-} &:= I_{|N_i^-|+1} \otimes \boldsymbol{P}_u \in \mathbb{R}^{2mH_p(|N_i^-|+1)\times mH_p(|N_i^-|+1)}, \\ \boldsymbol{U}^{i-}(k) &:= \left[\boldsymbol{U}_i^T, \boldsymbol{U}_{i1}^T, \boldsymbol{U}_{i2}^T, \cdots, \boldsymbol{U}_{i_{|N_i^-|}}^T\right]^T. \end{aligned}$$

Next, define $\boldsymbol{c}_j^{i-} := [1, 0, \cdots, 0, -1_{j-th}, 0, \cdots, 0]_{1\times(|N_i^-|+1)} \otimes \boldsymbol{e}_2 \otimes \boldsymbol{I}_m$ as the information collection vector, which helps agent i collect the information from its neighbor(s) in the screened neighbor set N_i^-. Here, the term $\otimes\boldsymbol{e}_2$ indicates that only the position state is used for optimization calculation. Then stack the information collection vectors in the subsystem S_i^- as

$$\boldsymbol{c}^{i-} := \left[\left(\boldsymbol{c}_{i1}^i\right)^T, \left(\boldsymbol{c}_{i2}^i\right)^T, \cdots, \left(\boldsymbol{c}_{i_{|N_i^-|}}^i\right)^T\right]^T \in \mathbb{R}^{m(|N_i^-|)\times 2m(|N_i^-|+1)}. \tag{14}$$

Meanwhile, define the desirable distance between agent i agent j as

$$\boldsymbol{l}_j^i := \frac{d\boldsymbol{q}_{ji}}{d_{ij}} = \frac{d\cdot\boldsymbol{e}_2\otimes\boldsymbol{I}_m(\boldsymbol{x}_i - \boldsymbol{x}_j)}{\|\boldsymbol{e}_2\otimes\boldsymbol{I}_m(\boldsymbol{x}_i - \boldsymbol{x}_j\|}. \tag{15}$$

Then, stack the desirable distances in the subsystem S_i^- as

$$\boldsymbol{l}^{i-} = \left[\left(\boldsymbol{l}_{i1}^i\right)^T, \left(\boldsymbol{l}_{i2}^i\right)^T, \cdots, \left(\boldsymbol{l}_{i_{|N_i^-|}}^i\right)^T \right]^T \in \mathbb{R}^{m(|N_i^-|)\times 1}. \tag{16}$$

Next, define

$$\boldsymbol{g}(\boldsymbol{q}^{i-}) = \boldsymbol{c}^{i-}\boldsymbol{x}^{i-} - \boldsymbol{l}^{i-}, \tag{17}$$

with $\boldsymbol{c}^{i-}$ and $\boldsymbol{l}^{i-}$ given in (14) and (16), respectively. Thus,

$$\|\boldsymbol{g}(\boldsymbol{q}^{i-})\|^2 = \sum_{i,j(j\in N_i^-)} \|\boldsymbol{q}_{ji} - d\boldsymbol{q}_{ji}/\|\boldsymbol{q}_{ji}\|\|^2 \tag{18}$$

collects the potential error between each edge length of screened subsystem S_i^- and the desirable distance d. Then, the predicted potential error of S_i^- in future H_p steps is $\boldsymbol{G}^{i-}(k+1|k) := \left[(\boldsymbol{g}^{i-}(k+1|k))^T, (\boldsymbol{g}^{i-}(k+2|k))^T, \cdots, (\boldsymbol{g}^{i-}(k+H_p|k))^T\right]^T$ and can be computed by

$$\boldsymbol{G}^{i-}(k+1|k) = \boldsymbol{C}^{i-}(k+1|k-1)\boldsymbol{X}^{i-}(k+1|k) - \boldsymbol{L}^{i-}(k+1|k-1), \tag{19}$$

where $\boldsymbol{C}^{i-}(k+1|k-1) := \left[(\boldsymbol{c}^{i-}(k+1|k-1))^T, (\boldsymbol{c}^{i-}(k+2|k-1))^T, \cdots, \left(\boldsymbol{c}^{i-}(k+H_p|k-1)\right)^T\right]^T$ and $\boldsymbol{L}^{i-}(k+1|k-1) := \left[(\boldsymbol{l}^{i-}(k+1|k-1))^T, (\boldsymbol{l}^{i-}(k+2|k-1))^T, \cdots, (\boldsymbol{l}^{i-}(k+H_p|k-1))^T\right]^T$.

Because the objective of flocking problem is to design a control law to form an α-lattice, we compute the control law $\boldsymbol{u}_i$ by solving the following finite horizon optimization problem

Problem $\mathcal{P}_i$: At instant k,

$$\begin{aligned}
\min_{\boldsymbol{U}^{i-}(k)} J_i(k, \boldsymbol{U}_{i-}(k)) &= \|\boldsymbol{G}^{i-}(k+1|k)\|^2 + \lambda\|\boldsymbol{U}^{i-}(k)\|^2 \\
&\overset{①}{=} \left\|\boldsymbol{C}^{i-}(k+1|k-1)\boldsymbol{X}^{i-}(k+1|k) - \boldsymbol{L}^{i-}(k+1|k-1)\right\|^2 + \lambda\left\|\boldsymbol{U}^{i-}(k)\right\|^2 \\
&\overset{②}{=} \left\|\boldsymbol{C}^{i-}(k+1|k-1)\left(\boldsymbol{P}_x^{i-}\boldsymbol{x}^{i-}(k) + \boldsymbol{P}_u^{i-}\boldsymbol{U}^{i-}(k)\right)\right. \\
&\quad \left. -\boldsymbol{L}^{i-}(k+1|k-1)\right\|^2 + \lambda\left\|\boldsymbol{U}^{i-}(k)\right\|^2 \\
&s.t. \quad \boldsymbol{x}_i(k) \in \mathbb{X} \\
&\qquad\ \ \boldsymbol{u}_i(k) \in \mathbb{U},
\end{aligned} \tag{20}$$

where $\overset{①}{=}$ and $\overset{②}{=}$ are guaranteed by Equations (19) and (13), respectively. Equation (20) can be converted into a quadratic programming problem. Particularly, if there exist no state and control input constraints, then optimize problem $\mathcal{P}_i$ by $\frac{\partial J_i(k,\boldsymbol{U}_{i-}(k))}{\partial \boldsymbol{U}_{i-}(k)} = 0$ to obtain the analytical solution

$$\boldsymbol{U}^{*i-}(k) = \left[\left(\boldsymbol{P}_u^{i-}\right)^T(\boldsymbol{C}^{i-})^T\boldsymbol{C}^{i-}\boldsymbol{P}_u^{i-} + \lambda\boldsymbol{I}\right]^{-1} \times \left(\boldsymbol{P}_u^{i-}\right)^T(\boldsymbol{C}^{i-})^T\left(\boldsymbol{C}^{i-}\boldsymbol{P}_x^{i-}\boldsymbol{x}^{i-}(k) - \boldsymbol{L}^{i-}\right). \tag{21}$$

First, pick agent i' optimal control input $\boldsymbol{U}_i^*(k)$ from the optimal control input sequence $\boldsymbol{U}^{*i-}(k)$ of the screened subsystem S_i^-.

$$\boldsymbol{U}_i^*(k) = [1, 0, \cdots, 0]_{1\times(|N_i^-|+1)} \otimes \boldsymbol{I}_{H_p} \otimes \boldsymbol{I}_m\boldsymbol{U}^{*i-}(k).$$

Usually, the first m entries of $\boldsymbol{U}_i^*(k)$ are extracted as the actual control input for agent i, namely,

$$\boldsymbol{u}_i^*(k|k) = [1, 0, \cdots, 0]_{1\times H_p} \otimes \boldsymbol{I}_m\boldsymbol{U}_i^*(k). \tag{22}$$

Remark 1
The reason why agent i uses the information at sampling instant $k-1$ to predict $\boldsymbol{C}^{i-}(k+1|k-1)$ and $\boldsymbol{L}^{i-}(k+1|k-1)$ in (19) is that agent i updates its current state $\boldsymbol{x}_i(k)$ with respect to the previous states of the subsystem S_i according to the MPC rule.

3.3. Distributed model predictive control flocking algorithm

Through solving the receding horizon optimization problem (20), we compute the distributed MPC law $\boldsymbol{u}_i^*(k|k)$ for each agent i at sampling instant k. Then, we propose the distributed MPC algorithm to achieve the flocks of the MAS.
Algorithm 1: *At instant* $k,\ \forall i \in \mathcal{V}$

(1) *Agent* i *monitors the information in the networked system to find its neighbors.*
(2) *Agent* i *determines its screened neighbor set* N_i^-, *collects the position states of the agents at the previous sampling instant* $k-1$ *in the screened subsystem, that is,* $\boldsymbol{x}_j(k-1), j \in S_i^-$ *and solves the receding horizon optimization problem (20) to obtain its local optimal control sequence* $\boldsymbol{U}_i^*(k)$ *and computes its state* $\boldsymbol{x}_i(k)$.
(3) *Agent* i *broadcasts its state* $\boldsymbol{x}_i(k)$ *in the networked system, facilitating other agents to solve their optimization problems in the next sampling instant* $k+1$.
(4) *Agent* i *extracts* $\boldsymbol{u}_i^*(k|k)$ *only from its local optimal control sequence* $\boldsymbol{U}_i^*(k)$.
(5) *Move horizon to the next sampling instant, set* $k = k+1$, *and return to step 1).*

Remark 2
Let us stress that, at the step (1) of Algorithm 1, 'monitor' does not mean that the agent needs to use the global information, because it only needs to make a judgment to determine its neighbors. Specifically, in actual situation, each agent aimlessly broadcasts its states and concurrently monitors the states of other agents in the environment. If the signal received by the agent is too weak to be utilized effectively, the agent will ignore this signal. In fact, the agent only cares about the signal, which it can use and regards this sender as its neighbor. On the other hand, the receiver also broadcasts its state, so the sender can simultaneously receive the information from the receiver, because all the agents are identical and they have the same capacities of broadcasting and monitoring. Therefore, the communication between a pair of neighboring agents is mutual and can be abstracted as the undirected topology. Generally, the intensity of the signal depends on the distance between the sender and the receiver, which explains why we define the communication range r_c as the neighbor selection criterion.

In Algorithm 1, each agent monitors the states including positions and velocities of other agents to find its neighbors, and then determines its screened neighbor set according to the rules in Section 2.2. And the purpose of collecting the position state is to minimize the error between the distance between each pair of neighboring agents and the desirable distance d by solving the receding horizon optimization problem in MPC. The proposed distributed MPC flocking algorithm is a synchronous parallel strategy, because all the agents in the networked system solve their optimization problems at each sampling instant.

4. STABILITY ANALYSES

Here, we present stability analysis of the distributed MPC flocking strategy by using the geometric properties of the optimal path followed by individual agents. We first give the definition of n-path and then two assumptions as follows:

Definition 1
[25] Given two points $P_1, P_2 \in \mathbb{R}^m$, let $\overline{P_1P_2}$ be the segment jointing them and denote by $|\overline{P_1P_2}|$ the segment length. An M-path is an ordered sequence of M points $T = \{P_1, P_2, \cdots, P_M\} \in \mathbb{R}^m$.

Assumption 1
There exists $\tau \geqslant 0$ such that for $\forall t > 0$, the network of the screened MAS governed by (9) is connected across $[t, t+\tau]$.

DOI: 10.1002/rnc

Assumption 2
For the screened MAS, given any initial position state $\boldsymbol{q}^{-}(0) \in \mathbb{R}^{Nm}$, there exists a nearest desired state $\boldsymbol{q}^{-0}$ satisfying $\|\boldsymbol{q}^{0}_{ji}\| = d, \forall (i,j) \in \mathcal{E}(\boldsymbol{q}^{-0})$, or $\boldsymbol{g}(\boldsymbol{q}^{-0}) = \boldsymbol{0}$ with

$$\|\boldsymbol{g}(\boldsymbol{q}^{-})\|^2 := \sum_{(i,j)\in\mathcal{E}(\boldsymbol{q}^{-})} \|\boldsymbol{q}_{ji} - d\boldsymbol{q}_{ji}/\|\boldsymbol{q}_{ji}\|\|^2. \tag{23}$$

Besides, for any $\boldsymbol{q}^{-1}, \boldsymbol{q}^{-2} \in \mathbb{R}^{Nm}$ with $\|\boldsymbol{q}^{-1} - \boldsymbol{q}^{-0}\| \leqslant \|\boldsymbol{q}^{-2} - \boldsymbol{q}^{-0}\|$, it always holds that $\|\boldsymbol{g}(\boldsymbol{q}^{-1})\| \leqslant \|\boldsymbol{g}(\boldsymbol{q}^{-2})\|$.

Remark 3
Here, $\mathcal{E}(\boldsymbol{q}^{-})$ indicates the edges of the screened MAS constructed by each agent i and its screened neighbors set N_i^{-}.

Next, a lemma [30] is presented to compare the segment lengths of different M-paths.

Lemma 1
Let $T_A = \{A_1, A_2, \cdots, A_M\}$ be an M-path. Given O, there always exists an M-path $T_B = \{B_1, B_2, \cdots, B_M\}$ with $B_1 = A_1$, pointing towards O and satisfying the following inequalities:

$$|\overline{B_j O}| \leqslant |\overline{A_j O}|, \tag{24}$$

$$|\overline{B_j B_{j+1}}| \leqslant |\overline{A_j A_{j+1}}|, \tag{25}$$

$$|\overline{B_{j+1} B_{j+2}} - \overline{B_j B_{j+1}}| \leqslant |\overline{A_{j+1} A_{j+2}} - \overline{A_j A_{j+1}}|, \tag{26}$$

with

$$|\overline{B_{M-1} B_M}| \leqslant |\overline{B_{M-2} B_{M-1}}|, \tag{27}$$

and $j = 1, 2, \cdots, M-2$.

Please refer to the papers [25, 30] for a proof of these results.

Theorem 1
Given Assumptions 1 and 2, the MAS (9) forms a rigid α-lattice flock by applying the distributed MPC law (22). Besides, if the initial state of the screened MAS $\boldsymbol{q}^{-0}$ satisfies $\|\boldsymbol{g}(\boldsymbol{q}^{-}(0)\| < d$, then no inter-agent collisions occur for all $t \geqslant 0$.

Proof
Part I (α-lattice flock): Given the MAS(9), a rigid α-lattice flock is achieved when

$$\begin{aligned} \lim_{k\to\infty} \|\boldsymbol{q}_j(k) - \boldsymbol{q}_i(k)\| = d, \forall (i,j) \in \mathcal{E}(\boldsymbol{q}), \\ \lim_{k\to\infty} \boldsymbol{p}_i(k) = \boldsymbol{p}_j(k), \forall i,j \in \mathbb{N}. \end{aligned} \tag{28}$$

Based on the Assumption 2, each screened subsystem S_i^{-} has a desired initial state $\boldsymbol{q}^{i-0}$ satisfying $\|\boldsymbol{q}_{ji,(j\in N_i^{-})}\| = d$, or $\boldsymbol{g}\left(\boldsymbol{q}^{i-0}\right) = \boldsymbol{0}$ because $\boldsymbol{q}^{i-0}$ is directly extracted by $\boldsymbol{q}^{-0}$. Besides, define $\boldsymbol{q}^{i-1}$ and $\boldsymbol{q}^{i-2}$ as two arbitrary distinct positions of screened subsystem S_i^{-}. According to Assumption 2, if $\left\|\boldsymbol{q}^{i-1} - \boldsymbol{q}^{i-0}\right\| \leqslant \left\|\boldsymbol{q}^{i-2} - \boldsymbol{q}^{i-0}\right\|$, then

$$\left\|\boldsymbol{g}\left(\boldsymbol{q}^{i-1}\right)\right\| \leqslant \left\|\boldsymbol{g}\left(\boldsymbol{q}^{i-2}\right)\right\|. \tag{29}$$

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Next, with index (18) and system dynamics (9), rewrite the optimization problem $J_i(k)$ of the screened subsystem S_i^- as

$$\begin{aligned}\min J_i(k) = \min \sum_{j=1}^{H_p} \left\| \boldsymbol{g}(\boldsymbol{q}^{i-}(k+j|k)) \right\|^2 \\ + \frac{\lambda}{T^4} \sum_{j=0}^{H_p-1} \|[\boldsymbol{q}_i(k+j+2|k) - \boldsymbol{q}_i(k+j+1|k)] \\ - [\boldsymbol{q}_i(k+j+1|k) - \boldsymbol{q}_i(k+j|k)]\|^2,\end{aligned} \tag{30}$$

with the last term following the fact that

$$\boldsymbol{u}_i(k+j|k) = \{[\boldsymbol{q}_i(k+j+2|k) - \boldsymbol{q}_i(k+j+1|k)] - [\boldsymbol{q}_i(k+j+1|k) - \boldsymbol{q}_i(k+j|k)]\}/T^2. \tag{31}$$

With Assumption 2 and Lemma 1, there always exists a position sequence $\left\{\boldsymbol{q}^{i-*}(k+1|k),\right.$ $\left.\boldsymbol{q}^{i-*}(k+2|k), \cdots, \boldsymbol{q}^{i-*}(k+H_p|k)\right\}$ pointing towards $\boldsymbol{q}^{i-0}$ $\left(\text{with } \boldsymbol{g}\left(\boldsymbol{q}^{i-0}\right) = \boldsymbol{0}\right)$ such that

$$\left\| \boldsymbol{q}^{i-*}(k+j|k) - \boldsymbol{q}^{i-0} \right\| \leqslant \left\| \boldsymbol{q}^{i-}(k+j|k) - \boldsymbol{q}^{i-0} \right\|, \tag{32}$$

$$\left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+j|k)\right) \right\| \leqslant \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-}(k+j|k)\right) \right\|, \tag{33}$$

$$\begin{aligned}&\left\| \left[\boldsymbol{q}^{i-*}(k+j+2|k) - \boldsymbol{q}^{i-*}(k+j+1|k)\right] - \left[\boldsymbol{q}^{i-*}(k+j+1|k) - \boldsymbol{q}^{i-*}(k+j|k)\right] \right\|, \\ &\leqslant \left\| \left[\boldsymbol{q}^{i-}(k+j+2|k) - \boldsymbol{q}^{i-}(k+j+1|k)\right] - \left[\boldsymbol{q}^{i-}(k+j+1|k) - \boldsymbol{q}^{i-}(k+j|k)\right] \right\|, \\ &\qquad j = 0, 1, \cdots, H_p - 1,\end{aligned} \tag{34}$$

and

$$\boldsymbol{q}^{i-*}(k+H_p-1|k) - \boldsymbol{q}^{i-*}(k+H_p-2|k) = \boldsymbol{q}^{i-*}(k+H_p|k) - \boldsymbol{q}^{i-*}(k+H_p-1|k). \tag{35}$$

In order to facilitate the theoretical analysis, we assume that agent $j \in N_i^-$ has zero accelerations in the prediction of agent i, that is, $\boldsymbol{u}_j(k+l|k) = 0,\ l = 0, 1, \cdots, H_p - 1$. This setting only affects the flocking speed, but will not influence the stability of the MAS (9) by applying the control law (22). Then submit (31) into (34) and (35) yields

$$\|\mathbf{u}_i^*(k+j|k)\| \leqslant \|\mathbf{u}_i(k+j|k)\|,\ j = 0, 1, \cdots, H_p - 1. \tag{36}$$

Also, from (33) and (34), the optimal position sequence of the optimization index (30) is necessary pointing towards $\boldsymbol{q}^{i-0}$, such that

$$\left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+j|k)\right) \right\| \leqslant \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+j-1|k)\right) \right\|, j = 1, 2, \cdots, H_p. \tag{37}$$

Then, define $\widetilde{\boldsymbol{U}}_i(k+1|k+1) = \left[(\boldsymbol{u}_i^*(k+1|k))^T, (\boldsymbol{u}_i^*(k+2|k))^T, \cdots, (\boldsymbol{u}_i^*(k+H_p-1|k))^T, \boldsymbol{0}^T\right]^T$. At time instant $k+1$, the best available value $\left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+1|k)\right) \right\|$ is obtained by applying $\boldsymbol{u}_i^*(k|k)$, so the position sequence of $\boldsymbol{q}^{i-}(k+j|k+1)$ starts at $\boldsymbol{q}^{i-*}(k+1|k)$. By prediction iteration, one has

DOI: 10.1002/rnc

$$
\begin{aligned}
J_i\left(k+1, \widetilde{\boldsymbol{U}}_i(k+1|k+1)\right) &= \left\| \boldsymbol{g}(\boldsymbol{q}^{i-*}(k+H_p+1|k)) \right\|^2 + \sum_{j=2}^{H_p} \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+j|k)\right) \right\|^2 \\
&\quad + \lambda \sum_{j=1}^{H_p-1} \|\boldsymbol{u}_i^*(k+j|k)\| \\
&\leqslant \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+H_p|k)\right) \right\|^2 + \sum_{j=2}^{H_p} \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+j|k)\right) \right\|^2 \\
&\quad + \lambda \sum_{j=1}^{H_p-1} \|\boldsymbol{u}_i^*(k+j|k)\|,
\end{aligned}
$$

where the '$\leqslant$' follows from Lemma 1. More specifically, the last component of $\widetilde{\boldsymbol{U}}_i(k+1|k+1)$ is $\mathbf{0}$ (or the velocity $\boldsymbol{p}_i(k+H_p-1|k)$ is assumed to be constant), which corresponds to (27). Considering that $\boldsymbol{q}^{i-*}(k+j|k),\ j=1,\cdots,H_p$ is pointing towards $\boldsymbol{q}^{i-0}$, one has $\left\| \boldsymbol{g}(\boldsymbol{q}^{i-*}(k+H_p+1|k)) \right\|^2 \leqslant \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+H_p|k)\right) \right\|^2$. Thus,

$$
\begin{aligned}
&J_i(k+1, \widetilde{\boldsymbol{U}}_i(k+1|k+1)) - J_i(k, \boldsymbol{U}_i^*(k|k)) \\
&\leqslant \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+H_p|k)\right) \right\|^2 - \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+1|k)\right) \right\|^2 - \lambda \|\boldsymbol{u}_i^*(k|k)\|^2 \leqslant 0.
\end{aligned} \tag{38}
$$

The last '$\leqslant$' is guaranteed by the inequality (37) together with $-\lambda\|\boldsymbol{u}_i^*(k|k)\|^2 \leqslant 0$. However, the $\widetilde{\boldsymbol{U}}_i(k+1|k+1)$ is not necessary the optimal control law $\widetilde{\boldsymbol{U}}_i^*(k+1|k+1)$ at time instant $k+1$, and thus,

$$
J_i(k+1, \widetilde{\boldsymbol{U}}_i^*(k+1|k+1)) \leqslant J_i(k+1, \widetilde{\boldsymbol{U}}_i(k+1|k+1)). \tag{39}
$$

Then, according to (38) and (39)[‡], one has

$$
J_i(k+1, \widetilde{\boldsymbol{U}}_i^*(k+1|k+1)) \leqslant J_i(k, \boldsymbol{U}_i^*(k|k)). \tag{40}
$$

and the '=' holds if and only if the control law $\boldsymbol{U}_i^*(k|k) = \mathbf{0}$. Taking $J_i(k, \widetilde{\boldsymbol{U}}_i(k|k))$ as Lyapunov function and considering the arbitrary selection of the screened subsystem S_i^-, one has that the screened MAS governed by (9) and (22) is stable with an optimal equilibrium at $\boldsymbol{q}^{i-0}$.

Then, given $J_i(k) \geqslant 0$ and (39), (40), one has

$$
\lim_{k\to\infty} J_i\left(k+1, \widetilde{\boldsymbol{U}}_i^*(k+1|k+1)\right) - J_i\left(k, \boldsymbol{U}_i^*(k|k)\right) = 0,
$$

and thus, it follows from (38) that

$$
\lim_{k\to\infty} \|\boldsymbol{u}_i^*(k|k)\| = 0, \tag{41}
$$

$$
\lim_{k\to\infty} \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+H_p|k)\right) \right\|^2 - \left\| \boldsymbol{g}\left(\boldsymbol{q}^{i-*}(k+1|k)\right) \right\|^2 = 0. \tag{42}
$$

The convergence of $\boldsymbol{p}_i$ is guaranteed by Equation (41), because

$$
\lim_{k\to\infty} \|\boldsymbol{p}_i(k+1) - \boldsymbol{p}_i(k)\| = \lim_{k\to\infty} T\,\|\boldsymbol{u}_i^*(k|k)\| = 0.
$$

[‡]Correction added on 23 August 2016, after first online publication: Citation of equations (39) and (40) has been corrected to (38) and (39), respectively.

Moreover, due to the arbitrary selection of the positive weighting factor λ, one has that $\left\|g\left(q^{i-*}(k+j|k)\right)\right\|$ decreases with j until arriving $\left\|g\left(q^{i-0}\right)\right\| = 0$, which means $\lim_{k\to\infty}\||q_i(k) - q_{j\in N_i^-}(k)\| = d$ and $\lim_{k\to\infty} p_i(k) = p_{j\in N_i^-}(k)$. Given Assumption 1, one has that the flock of the screened MAS is achieved. On the other hand, for each agent i' neighbors outside the screened neighbor set, they always have the relative motion, which facilitates the formation of the flock, or they have already achieved the suitable states, which the desirable flocking conformation needs. Therefore, one has that (28) holds or a rigid α-lattice of the MAS is formed.

Part II (Collision avoidance): Here, we prove this by contradiction. Considering $\|g(q^-(0))\| < d$, one has $\|g(q^{i-}(0))\| \leqslant \|g(q^-(0))\| < d$. Given (37), one obtains $\|g(q^{i-}(k+1))\| \leqslant \|g(q^{i-}(k))\| < d$, and thus,

$$\|g(q^{i-}(k))\| < d,\ \forall k > 0, \tag{43}$$

when $\|g(q^{i-}(0))\| < d$. Without loss of generality, assumes a collision happens between agent i and one of its screened neighbor $j \in N_i^-$ at time instant $k = \hat{k} > 0$, that is, $q_{ji}(\hat{k}) = 0$. Thus, one has

$$\|q_{ji} - dq_{ji}/\|q_{ji}\|\| = d,$$

and thus, $\left\|g\left(q^{i-}(\hat{k})\right)\right\| \geqslant d$, which contradicts (43). Therefore, collisions within the screened MAS are avoided. Besides, for each agent i' neighbors outside the screened neighbor set, they always have the relative motion, which facilitates the flocking process, and thus, there exists no collisions. To sum up, the collision avoidance of the MAS is guaranteed all along. □

5. SIMULATION RESULTS

In this section, a number of simulations are displayed to verify the effectiveness and advantage of the proposed distributed MPC algorithm.

Consider a MAS with $n = 50$ agents moving in a $x - y$ free space. Each agent follows dynamics (9) with distributed MPC input (22). Fifty agents, each endowed with a controller, are tasked to accomplish the lattice conformation where each pair of neighboring agents has the same distance. The initial planar positions and velocities of 50 agents: $q_i(0), p_i(0) \in \mathbb{R}^2, i = 1, 2, \cdots, 50$ are randomly chosen from the boxes: $[-15, 15] \times [-15, 15]$ and $[1, 2] \times [1, 2]$, respectively. Set the desirable distance $d = 7$, the allowable error $\delta = 0.1d$, the communication range $r_c = 1.2d$, the sampling interval $T = 0.1$ sec, and the prediction horizon $H_p = 3$.

The initial positions of the agents are shown in Figure 2(a) where the solid lines with arrows represent the velocity vectors. The movements and relationships of the agents are evolved from $k = 0$ to $k = 70$ are depicted throughout Figure 2. Fifty agents cooperatively achieve the geometry of flocks: quasi α-lattice as shown in Figure 2(f) where the solid lines among agents represent the neighboring relations. For convenience of viewing, the neighboring relations among agents are only depicted in Figure 2(f), in which each pair of neighboring agents has the approximately equal distance. The agents coordinate their distances and velocities according to the proposed distributed MPC algorithm. Namely, at each sampling instant, each agent monitors the information of other agents to find its neighbors or its subsystem, and then screens its neighbors via relative motion to determine the screened neighbor set for optimization.

We then use lattice disagreement index q_{err} and velocity disagreement index p_{err} to qualify the irregularity of an α-lattice structure and the dissensus of velocity, respectively. Here, define

$$q_{err}(k) = \frac{\sum_{i,j\in\mathcal{E}} \|d_{ij} - d\|}{|\mathcal{E}|}, \tag{44}$$

and

$$p_{err}(k) = \frac{\sum_{i,j\in\mathcal{V}} \|p_i(k) - \bar{p}_i(k)\|}{N}. \tag{45}$$

DOI: 10.1002/rnc

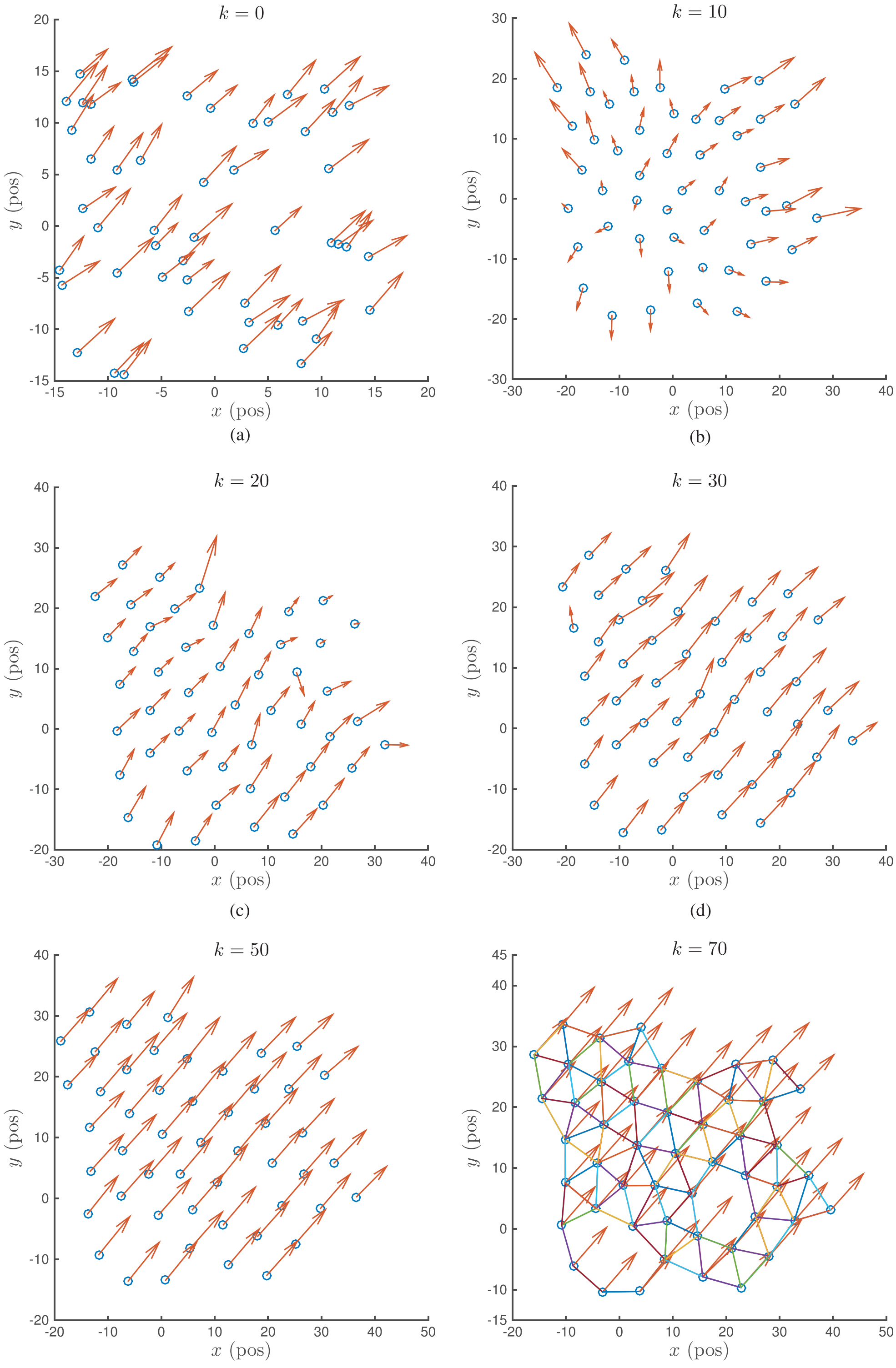

Figure 2. Flocking of 50 agents under proposed distributed model predictive control (MPC) strategy. [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 3(a) displays the trajectory of lattice disagreement index q_{err}, which asymptotically reaches a stable value(0.4052) that is less than the allowable error $\delta = 0.7(0.1d)$. Therefore, after certain steps, the formation of an α-lattice structure is guaranteed by our purposed control strategy.

DOI: 10.1002/rnc

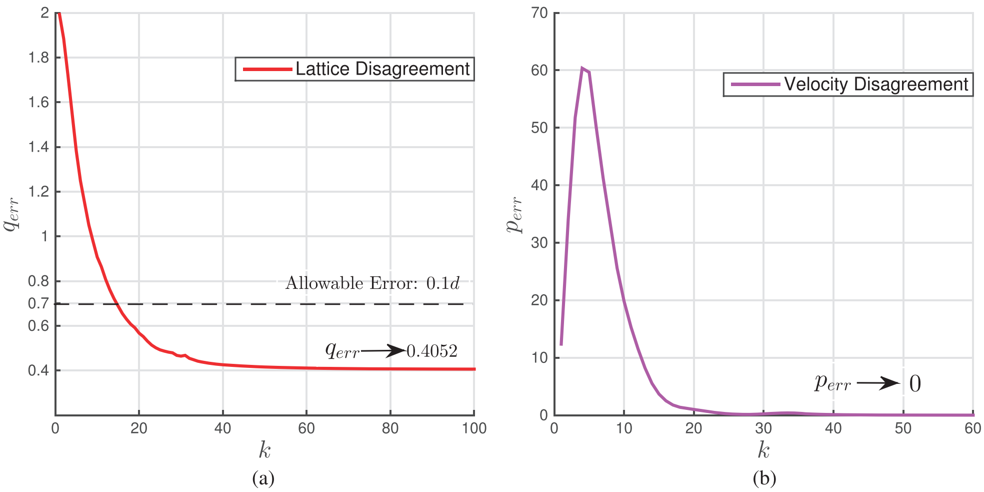

Figure 3. Trajectory evolution of lattice disagreement and velocity disagreement under proposed control strategy. [Colour figure can be viewed at wileyonlinelibrary.com]

Table I. The number of agent i' neighbor set N_i and its screened neighbor set N_i^- $(i = 1, 8, 17, 31, 40)$ at selected sampling instant $k = 1,\ 20,\ 50$.

	Agent 1		Agent 8		Agent 17		Agent 31		Agent 40	
Comparison	N_1	N_1^-	N_8	N_8^-	N_{17}	N_{17}^-	N_{31}	N_{31}^-	N_{40}	N_{40}^-
$k = 1$	16	14	7	6	11	8	4	3	20	18
$k = 20$	10	9	5	4	5	3	2	2	11	10
$k = 50$	6	4	4	4	4	3	2	2	5	5

Besides, the trajectory of velocity disagreement p_{err} is depicted in Figure 3(b) where the velocity consensus is achieved as indicated by p_{err} approaching to zero.

Table I and Figure 4 are depicted to visualize the performance of our proposed control strategy. Let us first stress that the MPC method has advantages in convergence efficacy of flocking by comparing the convergence speed of Olfati-Saber's flocking method [8] with Zhang *et al*. flock MPC [30] and the present distributed MPC strategy(22) in Figure 4. And this advantage has been explicitly described and illustrated in [30, 34]. Here, our proposed control strategy has two remarkable advantages: (i) Less local information is used to achieve the flock of the MAS via neighbor screening mechanism, that is, each agent screens its neighbor set and only collects the information of a part of its neighbors and itself to accomplish the optimization in proposed distributed MPC strategy, which is shown in Table I where $|N_i^-|$ is always less than or equal to $|N_i|$ $(i = 1, 8, 17, 31, 40)$ at arbitrarily selected sampling instant $k = 1,\ 20,\ 50$ and (ii) The convergence efficacy is substantially improved by distributed MPC method and neighbor screening mechanism as shown in Figure 4 where we compare the convergence speed of Olfati-Saber's flocking method [8] and Zhang *et al.* flock MPC [30] with our proposed control strategy.

We also recognize that the proposed control strategy still needs to be improved. In Figure 4(a), although the stable value of lattice disagreement by the present strategy is less than the allowable error $\delta = 0.7(0.1d)$, it is larger than the stable value by using Olfati-Saber's flocking method [8] or Zhang *et al.* flock MPC [30]. This means the quasi α-lattice structure obtained by our proposed strategy is less rigid. The possible reason is that less local information is used through the flocking process, and is thus ongoing research. But achieving the approximate flocks by using less neighbors' information has practical meanings when considering the constrained energy, delay in communication and computation capability in actual environment.

DOI: 10.1002/rnc

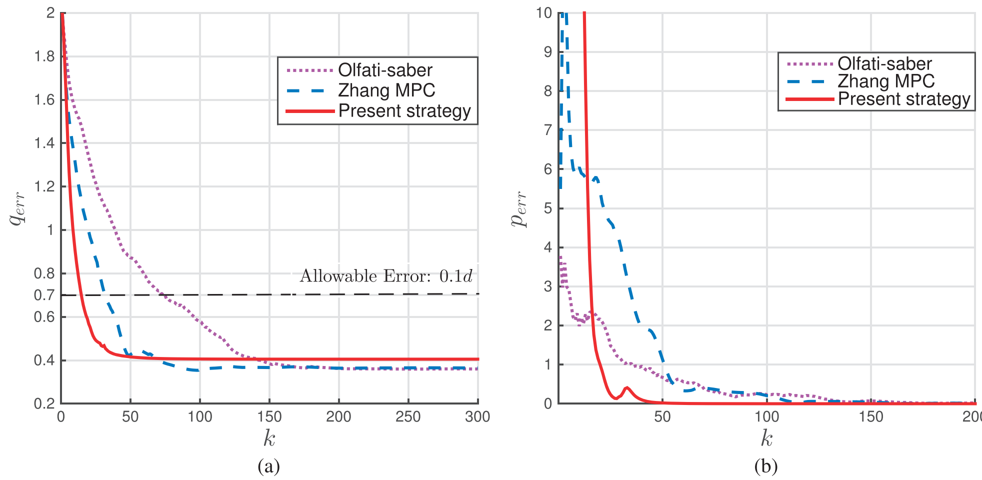

Figure 4. Comparison of trajectory evolution of lattice disagreement q_{err} and velocity disagreement p_{err} of the multi-agent system (MAS) under three different control protocols, that is, Olfati-Saber's flocking method [8], Zhang *et al*. flock model predictive control (MPC) [30] and the present strategy (22). [Colour figure can be viewed at wileyonlinelibrary.com]

6. CONCLUSION

In this paper, we have presented a distributed MPC strategy to achieve the flocks: quasi α-lattice structure via neighbor screening mechanism. At each sampling instant, all agents optimize their plans by screening their neighbors and collect the information in their screened subsystems for receding horizon optimization. We have theoretically guaranteed that the proposed control strategy leads to flock with inter-agent collision avoidance by using the geometric properties of the optimal path. The performance and the characteristic of proposed control strategy have been verified by a number of simulations. Open research topics to explore may include improving the rigidity of the flocks and taking practical applications into account such as goal seeking, obstacle avoidance, and multi-robot coordination.

ACKNOWLEDGEMENTS

This work was supported by the National Nature Science Foundation of China (61233004, 61590924, 61221003), the partly supported by the National Basic Research Program of China (973 Program-2013CB035500). The authors are grateful to the editor and the anonymous reviewers for their helpful comments and constructive suggestions with regard to the revision of the paper.

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DOI: 10.1002/rnc